

1.[2 points] Find the equation of the tangent line to the curve $y = \ln(x^2 - 3)$ at the point $(2, 0)$.

- A.** $y = x - 2$ **B.** $y = 4x - 8$ **C.** $y = 2x - 4$
D. $y = -3x + 6$ **E.** $y = 0$ **F.** $y = \frac{1}{2}x - 1$

2.[2 points] Find the slope of the tangent line to the curve $x^2y^2 + xy = 2$ at $(1, 1)$.

- A.** 2.01 **B.** -2.01 **C.** 1 **D.** -1 **E.** 0 **F.** 3.14

3.[2 points] Suppose $F(x) = f(g(x))$, $g(1) = 3$, $g'(1) = 4$, $f'(1) = 6$, $f'(3) = 5$. Find $F'(1)$.

A. 20 **B.** 24 **C.** 12 **D.** 30 **E.** 18 **F.** 15

4.[2 points] The rate of change of the radius of a balloon is 2 cm/min. Assuming that the balloon is a perfect sphere (and given that the volume of a sphere is $V = \frac{4}{3}\pi r^3$), what is the rate of change of the volume of the balloon (in cm^3/min) when the radius is 5 cm?

A. π **B.** 200π **C.** 8π **D.** $\frac{200}{3}\pi$ **E.** $\frac{4}{3}\pi$ **F.** 25π

5.[2 points] On what subinterval(s) of $[0, 2\pi]$ is the function $y = 3 \cos(2x) + 4x$ concave up?

6.[2 points] Find the derivative of $f(x) = \arcsin(3x^2) \ln(x)$.

7.[2 points] Newton's Method is being used to find a root of $f(x) = x^3 - 3x + 1$ with $x_1 = 0$. What is the value of x_3 ?

- A.** 0.3333 **B.** -1.0694 **C.** 0.3472
D. -0.4028 **E.** -0.3333 **F.** 0.3519

8.[2 points] Albert and Gertrude start walking from the same point. Albert walks due north at a rate of 4m/s, while Gertrude walks due east at a speed of 3m/s. At what rate is the distance between Albert and Gertrude increasing 30 seconds later?

- A.** 5.86m/s **B.** 12.12m/s **C.** 13.67m/s
D. 16.83m/s **E.** 5m/s **F.** 0.68m/s

9.[2 points] Which of the following statements is true about the graph of the function $f(x) = x^2e^{-x}$ on the interval $[1, 2]$?

- A. It is increasing and concave up.
- B. It is decreasing and concave up.
- C. It is increasing and concave down.
- D. It is decreasing and concave down.
- E. It is increasing and changes concavity.
- F. It is decreasing and changes concavity.

10.[2 points] If $\int_{-1}^5 f(x) \, dx = 3.7$ and $\int_{-1}^3 f(x) \, dx = 3.2$, what is the value of $\int_3^5 (3f(x) + 2x) \, dx$?

- | | | |
|---------|---------|------------------|
| A: 16.5 | B: 14.9 | C: 17.5 |
| D: 16.0 | E: 12.7 | F: none of these |

11.[2 points] What is the absolute maximum value attained by the function $f(x) = x^3 - 6x^2 + 9x - 6$ on the interval $[0, 4]$?

12.[2 points] Find $F''(1)$ if $F(x) = \int_1^x f(t) dt$ and $f(t) = \int_1^{2t} \sqrt{1+u^3} du$.

A. 0 **B.** $\sqrt{2}$ **C.** 3 **D.** 6 **E.** $\sqrt{8}$ **F.** 9

13.[2 points] Evaluate $\int_1^e \frac{dx}{x\sqrt{1+\ln x}}$.

- A. $\ln(1+e)$ B. $e \ln(1+e)$ C. $2\sqrt{2}-2$
D. $2e^{1/2}-e$ E. $2e^{1/2}-2$ F. $\frac{1}{\sqrt{1+e}}$

14.[2 points] Estimate the value of $\int_1^2 \cos(\ln(x)) dx$ by subdividing $[1, 2]$ into 4 small intervals and using the **midpoint method**.

15.[2 points] Evaluate the integral $\int \frac{1+2x}{\sqrt{1-x^2}} dx$.

Answer:

16.[2 points] Evaluate the integral $\int x^2 \ln x \, dx$.

Answer:

17.[2 points] Find $\int \frac{x^4}{x+1} dx$.

18.[2 points] Evaluate the integral $\int x \cos\left(3x + \frac{\pi}{4}\right) dx$.

Answer:

19.[2 points] Evaluate the following integral.

$$\int \frac{3x + 29}{x^2 + x - 6} dx.$$

20.[2 points] Let $y = x^{\tan(x)}$. Find dy/dx .

Answer:

21.[5 points] A piece of wire 1 m long is cut into two pieces to make a square and a circle. How should the wire be cut so that the total area enclosed is (a) a minimum and (b) a maximum? *Provide a detailed solution and draw a box around each answer.*

22.[5 points] Analyze the graph of the $f(x) = e^{-x} \sin x$ for $0 \leq x \leq 2\pi$. Label local maxima/minima and inflection points (with their coordinates) *on your graph*. Indicate concavity (up/down).

23.[5 points] Identify each of the following statements as either true or false:

1. the inverse function of $f(x) = \ln(3x)$ is $f^{-1}(x) = e^{-3x}$
2. the inverse function of $f(x) = \sin(x)$ is $f^{-1}(x) = \csc(x)$
3. an inflection point of a curve is where the second derivative is zero
4. a critical point of a curve is where the derivative is either zero or does not exist
5. the extreme value theorem says that a function $f(x)$ defined on an interval $[a, b]$ will attain its absolute maximum and absolute minimum at the critical points of $f(x)$
6. the Fundamental Theorem of Calculus says that $\int_a^b g'(x) dx = g(x)$
7. the Fundamental Theorem of Calculus says that $\frac{d}{dx} \int_a^b g(x) dx = g(x)$
8. the formula for the Riemann sum which estimates $\int_0^1 f(x) dx$ using the right endpoint rule is $\sum_{i=1}^n f(x_i) \delta x$ where $x_i = i(\delta x)$ and $\delta x = \frac{1}{n}$
9. if $h(x) = g(x) + 4$ and $g'(x) = f(x)$, then $h'(x) = f(x)$
10. if a is a critical point of f such that $f'(a) = 0$ and $f''(a) > 0$ then a is a local minimum of f